CW# 4

Math 3

Name: KEY

Practice Packet:
Inverses
Unit 6 (Chapter 5)

Date: _____

Period: A1 A2 A3 B1 B2 B3

SET 1:

1. (REVIEW) Rewrite or simplify each expression so that there are no rational (fraction) exponents.

   a. $(n^4)^{\frac{3}{2}}$ $4 \cdot \frac{3}{2} = 6$

   n^6

   b. $(27p^6)^{\frac{5}{3}}$ $6 \cdot \frac{5}{3} = 10$

   $27^{5/3}(p^6)^{5/3}$

   $(\sqrt[3]{27})^5 \cdot p^{10} \rightarrow 3^5 p^{10} \rightarrow 243p^{10}$

2. (NEW) Make a table of $y = \frac{1}{2}x - 3$ and its inverse function.

x	y
-4	-5
-2	-4
0	-3
2	-2
4	-1
6	0

x	y

   a. What is the relationship between the table for the original function and its inverse?

   b. Graph $y = \frac{1}{2}x - 3$ and its inverse function on the same set of axes.

   c. What is the equation of the inverse function?

$x = \frac{1}{2}y - 3$

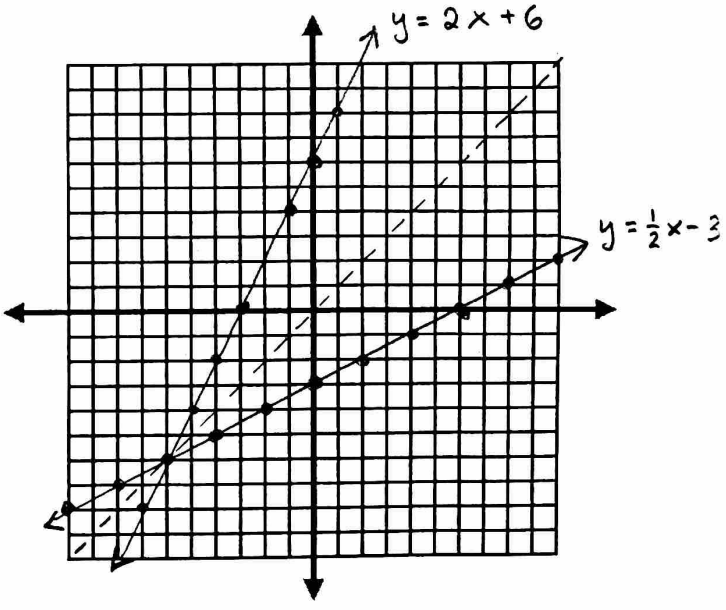
$+3 \quad +3$

$x + 3 = \frac{1}{2}y$

$2(x+3) = 2 \cdot \frac{1}{2}y$

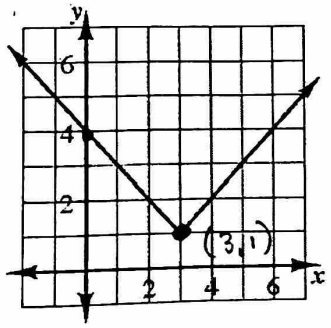
$y = 2(x+3)$

$y = 2x + 6$



   d. Does the graph of the two lines have a line of symmetry? If so, what is the equation of the line of symmetry?

Yes! $y = x$



3. (REVIEW) Write an equation to represent the graph at left.

Absolute value.
vertex at (3,1).
$h = 3 \quad k = 1$

$y = |x - 3| + 1$

No stretch or compression
$a = 1$

Check (0,4)

$4 = a|0 - 3| + 1$

$4 = a|-3| + 1$

$4 = a \cdot 3 + 1$

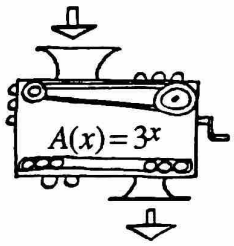
$4 - 1 = 3a + 1 - 1$

$\frac{3}{3} = \frac{3a}{3}$

$a = 1$

SET 2:

4. (REVIEW) Antonio's function machine is shown at right.



a. What is $A(2)$?

$A(2) = 3^2 = 9$

b. If 81 came out, what was dropped in?

$A(x) = 81 \rightarrow 81 = 3^x$

$x = 4$ because $3^4 = 81$

c. If 8 came out, what was dropped in? Be accurate to two decimal places.

$8 = 3^x$ $3^1 = 3$ $3^{1.9} = 8.06$ $3^{1.89} = 7.97$

$3^2 = 9$ $3^{1.8} = 7.22$ $x = 1.89$ is closest.

5. (EXPLORE) Visualize a sofa or couch you know well. If you cut it right down the middle vertically, what will the cross-section look like?

Answers vary...

6. (REVIEW) Solve the following equations, if possible. Some you can solve exactly, others approximately. If a solution is not possible, explain how you know.

a. $1^x = 5$

no solution!

Any exponent on 1 is still 1

b. $\sqrt{27^x} = 81$

$(\sqrt{27^x})^2 = 81^2$

$27^x = 81^2$

$(3^3)^x = (3^4)^2$

$3^{3x} = 3^8$

$3x = 8$

$x = \frac{8}{3}$

c. $2^x = 9$

$2^3 = 8$ → too low

$2^4 = 16$ → too high

$2^{3.5} = 11.3$ → too high

$2^{3.1} = 8.57$

$2^{3.2} = 9.19$

$x = \sim 3.2$

d. $25^{(x+1)} = 125^x$

$(5^2)^{(x+1)} = (5^3)^x$

$2x + 2 = 3x$

$-2x \quad -2x$

$x = 2$

e. $8^x = 2^5 \cdot 4^4$

$(2^3)^x = 2^5 (2^2)^4$

$2^{3x} = 2^5 \cdot 2^8$

$2^{3x} = 2^{13}$

$3x = 13$

$x = \frac{13}{3}$

7. (REVIEW) Graph the inequalities $y \geq |x-2|$ and $y \leq 4 - |x|$.

What is the area of the enclosed region? $A = 6$ units^2 (I counted the squares.)

test (0,0)

$0 \geq |0-2|$

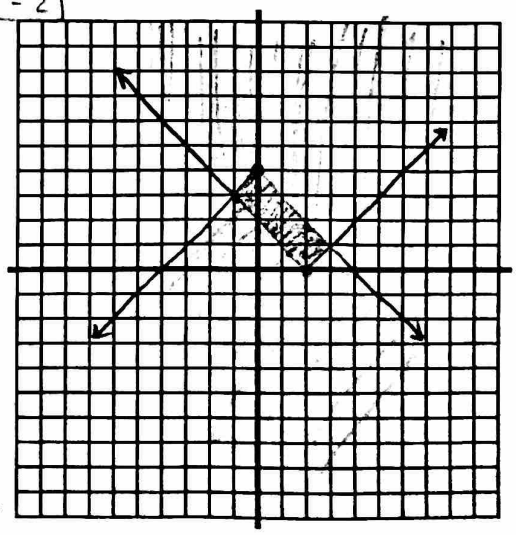
$0 \geq |-2|$

$0 \geq 2$ false

$0 \leq 4 - |0|$

$0 \leq 4$

true

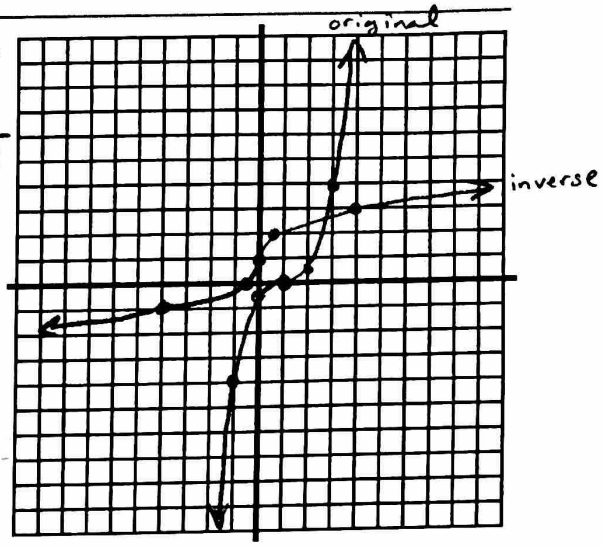


SET 3:

8. (NEW) Graph $y = \frac{1}{2}(x-1)^3$ and its inverse on the same set of axes.

x	-1	0	1	2	3
y	-4	-½	0	½	4

x	-4	-½	0	½	4
y	-1	0	1	2	3



9. (NEW) Write the equation of the inverse for each of the following functions.

a. $f(x) = 3x - 8$

Do ↓	
·3	÷3
-8	+8
	undo ↑

$f^{-1}(x) = \frac{x+8}{3}$

b. $f(x) = \frac{1}{2}x + 6$

$x = \frac{1}{2}y + 6$

$-6 \quad -6$

$x - 6 = \frac{1}{2}y$

$2(x-6) = 2 \cdot \frac{1}{2}y$ → $2x - 6 = y$

$f^{-1}(x) = 2x - 6$

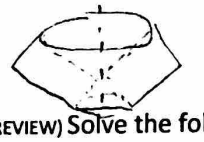
c. $f(x) = \frac{x+6}{2}$

Do ↓	
+6	-6
÷2	·2
	undo ↑

$f^{-1}(x) = 2(x-6)$ $\quad f^{-1}(x) = 2x - 12$

10. (EXPLORE) Mercucio believes that whenever a rectangle is rotated about a line, the result is always a cylinder. Do you agree or disagree? Justify your answer completely, explaining your decision.

Disagree! If the rectangle doesn't share a side w/ the line of rotation...




etc.

11. (REVIEW) Solve the following systems of equations. In other words, find values of a and b that make each system true. Be sure to show your work or explain your thinking clearly.

a. $3 = a \cdot b^0$

$75 = a \cdot b^2$

$3 = a \cdot 1$

$a = 3$

$\frac{75}{3} = \frac{3 \cdot b^2}{3}$

$25 = b^2$

$\sqrt{25} = \sqrt{b^2}$

$5 = b$

b. $18 = a \cdot b^2$

$54 = a \cdot b^3$

$\frac{18}{b^2} = \frac{a \cdot b^2}{b^2}$

$a = \frac{18}{b^2}$

$54 = \frac{18}{b^2} \cdot b^3$

$54 = \frac{18b^3}{b^2}$

$\frac{54}{18} = \frac{18b}{18}$

$3 = b$

$a = \frac{18}{(3)^2}$

$a = \frac{18}{9}$

$a = 2$

12. (REVIEW) A circle has the equation $x^2 + (y+2)^2 = r^2$. If the circle is shifted 2 units to the left, 5 units up, and the radius is doubled, what will its new equation be?

$(x+2)^2 + (y-3)^2 = 4r^2$

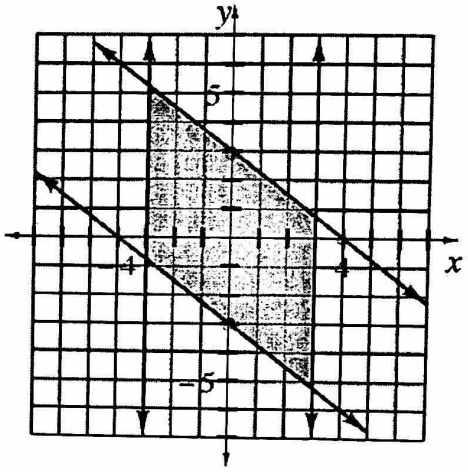
Substitute $(2r)^2$ for $(r)^2$

old center $(0, -2)$

new center $(-2, 3)$

SET 4:

13. (REVIEW) Write a system of inequalities that could be represented by the graph at right.



$y \leq -\frac{3}{4}x + 3$

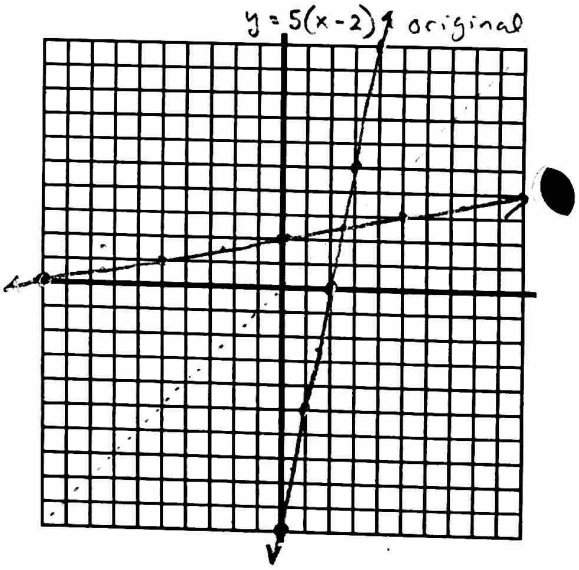
$y \geq -\frac{3}{4}x - 3$

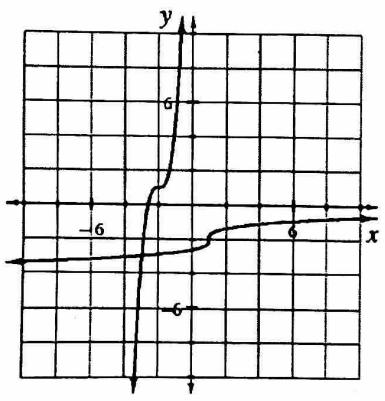
$y \geq -3$

$y \leq 3$

14. (NEW) Graph $y = 5(x-2)$ and its inverse on the same set of axes. Label the graph and the inverse function with their equations.

Do ↓	
-2	+2
·5	÷5
undo ↓	



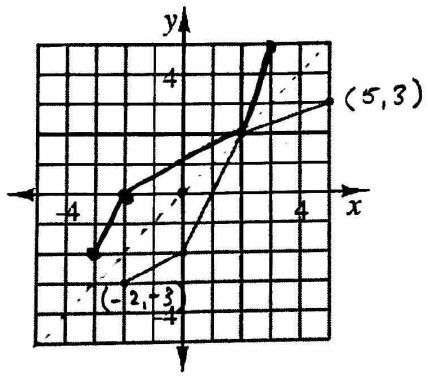


15. (NEW) Look at the graph of a function and its inverse, shown at left. If $h(x)$ is a function and $h^{-1}(x)$ is its inverse, can you tell which is which? Why or why not?

No you can't tell. They're both inverses of each other.

. . .

16. (NEW) The function *f(x)* is represented in the graph at right. Draw a graph of its inverse function. Be sure to state the domain and range for both *f(x)* and $f^{-1}(x)$.



$f(x)$
D: $-2 \le x \le 5$
R: $-3 \le y \le 3$

$f^{-1}(x)$
D: $-3 \le x \le 3$
R: $-2 \le y \le 5$

What is the relationship between the domain and range of the original function and the domain and range of the inverse?

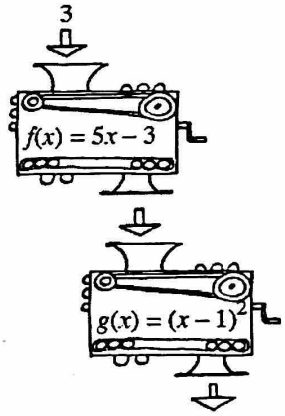
They switch.

17. (NEW) Trejo says that if you know the x-intercepts, y-intercepts, domain, and range of a function then you automatically know the x-intercepts, y-intercepts, domain, and range of its inverse. Hilary disagrees. She says you know the intercepts but that is all you know for sure. Who is correct? Justify your answer.

The x int of $f(x)$ become the y-int of $f^{-1}(x)$
The y-int of $f(x)$ become the x-int of $f^{-1}(x)$
Trejo is right about D and R too.

SET 5:

18. (EXPLORE) Two function machines, $f(x)=5x-3$ and $g(x)=(x-1)^2$, are shown at right.



a. Suppose f(x), is dropped into the g(x) machine. That is, 5x - 3 will be the input of the g(x) machine. This is written as g(f(x)). What is this output?

$g(5x-3) = (5x-3-1)^2 = g(f(x)) = (5x-4)^2$

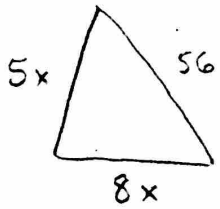
b. Using the same function machines, what is g(f(3))? That is, what is the output of g(x) when the input is f(3)?

$g(f(3)) = (5(3)-4)^2 = (15-4)^2 = (11)^2 = 121$

c. Using the same function machines, what is f(g(3))? Be careful! The result is different from the last one because the *order* in which you use the machines has been switched!

$f(g(x)) = 5(x-1)^2 - 3$
$f(g(3)) = 5(3-1)^2 - 3 = 5(2)^2 - 3 = 5(4) - 3 = 20 - 3 = 17$

19. (EXPLORE) The 160 cm border for a triangular tail fin of a space vehicle is built from small ceramic rods. Each side of the triangular tail is built using several small rods of equal length. One side of the tail fin must be exactly 56 cm long. So that ceramic rods do not have to be broken to fit the border and to meet aerodynamic constraints, the length of the other two sides must be in a ratio of 5:8. What are the lengths of the three sides of the tail fin?



$P = 160$ cm

$5x + 8x + 56 = 160$
$-56 \quad +56$
$\frac{13x}{13} = \frac{104}{13}$
$x = 8$

Lengths
$5(8) = 40$ cm
$8(8) = 64$ cm
given 56 cm

20. (REVIEW & EXPLORE) To what power do you have to raise:

a. 3 to get 27? (3)
$3^3 = 27$

b. 2 to get 32? (5)
$2^5 = 32$

c. 5 to get 625?
$5^4 = 625$ (4)

d. 64 to get 8? (1/2)
$\sqrt{64} = 64^{1/2} = 8$

e. 81 to get 3? (1/4)
$\sqrt[4]{81} = 81^{1/4} = 3$

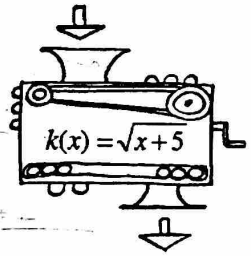
f. 64 to get 2?
$\sqrt[6]{64} = 64^{1/6} = 2$ (1/6)

g. (x^2) to get x^1?
(1/2) $(x^2)^{1/2} = x^1$

h. (x^3) to get x^{12}?
$(x^3)^4 = x^{12}$ (4)

i. x to get x^a
$x^a = x^a$ (a)

21. (NEW) Kirsta was working with the function machine shown at right, but when she turned her back, her little brother Caleb dropped in a number. She did not see what he dropped in, but she did see what fell out: 9.



a. What operations must she perform on 9 to undo what her machine did? Use this to determine what number Caleb dropped in.

Square it, then subtract 5.

$(9)^2 = 81$

$81 - 5 = 76$

b. Write an equation for a machine that will undo Kirsta's machine.

$k^{-1}(x) = x^2 - 5$

22. (REVIEW) Graph the systems below.

a. $y \leq -4x + 5$ → $0 \leq -4(0) + 5$ TRUE

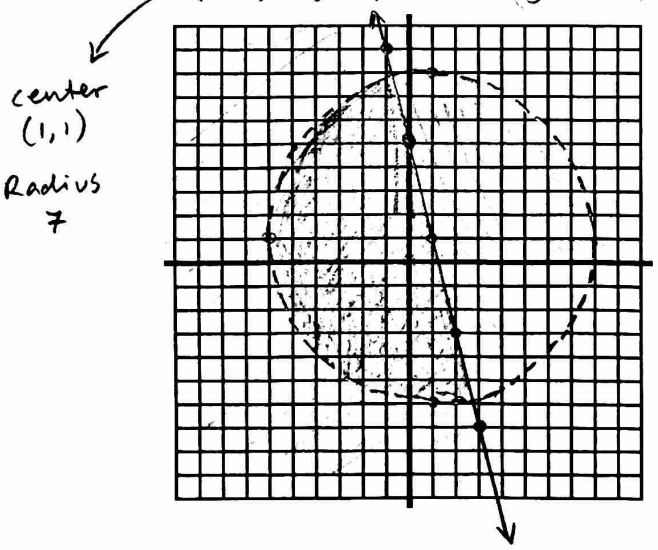
$(x-1)^2 + (y-1)^2 < 49$ → $(0-1)^2 + (0-1)^2 < 49$

$2 < 49$

TRUE

center (1, 1)

Radius 7



b. $1 + x - y > 3x - 2y - 4$ $y = 2x - 5$

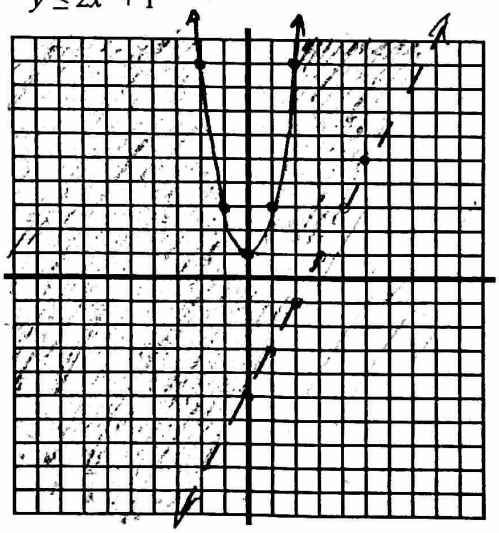
$y \leq 2x^2 + 1$

$1 + x - y = 3x - 2y$

$+2y \quad +2y$

$1 + x + y = 3x - 4$

$-1 \quad -x \qquad -x \quad -1$



Check (0, 0)

$1 + 0 - 0 > 3(0) - 2(0) - 4$

$1 > -4$ TRUE

$0 \leq 2(0)^2 + 1$

$0 \leq 1$ TRUE